Geometric Progression

Question 1.
Find, which of the following sequence form a G.P. :
(i) 8, 24, 72, 216, ……
(ii) $\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}$, ……..
(iii) 9, 12, 16, 24, ……

Solution 1(i).

Given sequence: 8, 24, 72, 216………
Now,
$$\frac{24}{8} = 3, \quad \frac{72}{24} = 3, \quad \frac{216}{72} = 3$$
Since $\frac{24}{8} = \frac{72}{24} = \frac{216}{72} = \ldots\ldots = 3$, the given sequence is a G.P. with common ratio 3.

Solution 1(ii).

Given sequence: $\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}$………
Now,
$$\frac{1/24}{1/8} = \frac{1}{3}, \quad \frac{1/72}{1/24} = \frac{1}{3}, \quad \frac{1/216}{1/72} = \frac{1}{3}$$
Since $\frac{1/24}{1/8} = \frac{1/72}{1/24} = \frac{1/216}{1/72} = \ldots\ldots = \frac{1}{3}$, the given sequence is a G.P. with common ratio $\frac{1}{3}$.

Solution 1(iii).

Given sequence: 9, 12, 16, 24………
Now,
$$\frac{12}{9} = \frac{4}{3}, \quad \frac{16}{12} = \frac{4}{3}, \quad \frac{24}{16} = \frac{3}{2}$$
Since $\frac{24}{8} = \frac{72}{24} \neq \frac{216}{72}$, the given sequence is not a G.P.

Question 2.
Find the 9th term of the series :
1, 4, 16, 64 ....

Solution:

Given sequence: 1, 4, 16, 64.........

Now,

$\frac{4}{1} = 4, \ \frac{16}{4} = 4, \ \frac{64}{16} = 4$

Since $\frac{4}{1} = \frac{16}{4} = \frac{64}{16} = \ldots\ldots = 4$, the given sequence is a G.P.

with first term, a = 1 and common ratio, r = 4.

Now, $t_n = ar^{n-1}$

$\Rightarrow t_9 = 1 \times 4^8 = 65536$

Question 3.
Find the seventh term of the G.P. :
1, $\sqrt{3}$, 3, $3\sqrt{3}$ .....

Solution:

Given G.P.: 1, $\sqrt{3}$, 3, $3\sqrt{3}$, ...

Here,

First term, a = 1

Common ration, $r = \frac{\sqrt{3}}{1} = \sqrt{3}$

Now, $t_n = ar^{n-1}$

$\Rightarrow t_7 = 1 \times \left(\sqrt{3}\right)^6 = 27$

Question 4.

Find the 8th term of the sequence :
$\frac{3}{4}, 1\frac{1}{2}$ 3, ...

Solution:

Given sequence: $\frac{3}{4}, 1\frac{1}{2}, 3, \ldots\ldots$

i.e. $\frac{3}{4}, \frac{3}{2}, 3, \ldots..$

Now,

$\frac{3/2}{3/4} = 2, \quad \frac{3}{3/2} = 2,$

Since $\frac{3/2}{3/4} = \frac{3}{3/2} = \ldots\ldots = 2,$ the given sequence is a G.P.

with first term, $a = \frac{3}{4}$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$\Rightarrow t_8 = \frac{3}{4} \times 2^7 = \frac{3}{4} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^5 = 96$

Question 5.

Find the 10th term of the G.P. :

Solution:

Given G.P.: $12, 4, 1\frac{1}{3}, \ldots\ldots.$

Here,

First term, $a = 12$

Common ration, $r = \frac{4}{12} = \frac{1}{3}$

Now, $t_n = ar^{n-1}$

$\Rightarrow t_{10} = 12 \times \left(\frac{1}{3}\right)^9 = 12 \times \frac{1}{19683} = \frac{4}{6561}$

Question 6.
Find the nth term of the series :

Solution:

Given series: 1, 2, 4, 8, ...

Now,

$\frac{2}{1} = 2, \ \frac{4}{2} = 2, \ \frac{8}{4} = 2$

Since $\frac{2}{1} = \frac{4}{2} = \frac{8}{4} = \ldots\ldots = 2$, the given sequence is a G.P.

with first term, a = 1 and common ratio, r = 2.

Now, $t_n = ar^{n-1}$

$\Rightarrow t_n = 1 \times 2^{n-1} = 2^{n-1}$

Question 7.

Find the next three terms of the sequence :

$\sqrt{5}$, 5, $5\sqrt{5}$, ...

Solution:

Given sequence: $\sqrt{5}$, 5, $5\sqrt{5}$,........

Now,

$\frac{5}{\sqrt{5}} = \sqrt{5}, \ \frac{5\sqrt{5}}{5} = \sqrt{5}$

Since $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{52} = \ldots\ldots = \sqrt{5}$, the given sequence is a G.P.

with first term, $a = \sqrt{5}$ and common ratio, $r = \sqrt{5}$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next three terms:

4^{th} term $= \sqrt{5} \times \left(\sqrt{5}\right)^3 = \sqrt{5} \times 5\sqrt{5} = 25$

5^{th} term $= \sqrt{5} \times \left(\sqrt{5}\right)^4 = \sqrt{5} \times 25 = 25\sqrt{5}$

6^{th} term $= \sqrt{5} \times \left(\sqrt{5}\right)^5 = \sqrt{5} \times 25\sqrt{5} = 125$

Question 8.

Find the sixth term of the series :

$2^2, 2^3, 2^4$, ...

Solution:

Given sequence: $2^2, 2^3, 2^4, \ldots..$

Now,

$\frac{2^3}{2^2} = 2, \quad \frac{2^4}{2^3} = 2$

Since $\frac{2^3}{2^2} = \frac{2^4}{2^3} = \ldots\ldots = 2$, the given sequence is a G.P.

with first term, $a = 2^2 = 4$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$\therefore \ t_6 = 4 \times (2)^5 = 4 \times 32 = 128$

Question 9.

Find the seventh term of the G.P. :
[late]\sqrt{3}+1,1, \frac{\sqrt{3}-1}{2}[/latex], ……………..

Solution:

Given G.P.: $\sqrt{3}+1, 1, \frac{\sqrt{3}-1}{2}, \ldots\ldots$

Here,

First term, $a = \sqrt{3}+1$

Common ration, $r = \frac{1}{\sqrt{3}+1}$

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_7 = \left(\sqrt{3}+1\right) \times \left(\frac{1}{\sqrt{3}+1}\right)^6$$

$$= \left(\frac{1}{\sqrt{3}+1}\right)^5$$

$$= \left(\frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}\right)^5$$

$$= \left(\frac{\sqrt{3}-1}{2}\right)^5$$

$$= \frac{1}{32}\left(\sqrt{3}-1\right)^5$$

Question 10.
Find the G.P. whose first term is 64 and next term is 32.

Solution:

First term, $a = 64$

Second term, $t_2 = 32$

$\Rightarrow ar = 32$

$\Rightarrow 64 \times r = 32$

$\Rightarrow r = \frac{32}{64} = \frac{1}{2}$

$\therefore$ Required G.P. $= a, ar, ar^{n-1}, ar^{n-2}, \ldots\ldots$

$$= 64, 32, 64 \times \left(\frac{1}{2}\right)^2, 64 \times \left(\frac{1}{2}\right)^3, \ldots\ldots$$

$$= 64, 32, 16, 8, \ldots\ldots$$

Question 11.
Find the next three terms of the series:

$\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots\ldots\ldots$

Solution:

Given sequence: $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots$

Now,

$$\frac{2/9}{2/27} = 3, \quad \frac{2/3}{2/9} = 3$$

Since $\frac{2/9}{2/27} = \frac{2/3}{2/9} = \ldots\ldots = 3$, the given sequence is a G.P.

with first term, $a = \frac{2}{27}$ and common ratio, $r = 3$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next three terms:

4^{th} term $= \frac{2}{27} \times (3)^3 = \frac{2}{27} \times 27 = 2$

5^{th} term $= \frac{2}{27} \times (3)^4 = \frac{2}{27} \times 27 \times 3 = 6$

6^{th} term $= \frac{2}{27} \times (3)^5 = \frac{2}{27} \times 27 \times 9 = 18$

Question 12.
Find the next two terms of the series
2 − 6 + 18 − 54 …………

Solution:

Given series: $2 - 6 + 18 - 54 \ldots\ldots$

Now,

$\frac{-6}{2} = -3, \quad \frac{18}{-6} = -3, \quad \frac{-54}{18} = -3$

Since $\frac{-6}{2} = \frac{18}{-6} = \frac{-54}{18} = \ldots\ldots = -3,$ the given sequence is a G.P.

with first term, $a = 2$ and common ratio, $r = -3$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next two terms:

5^{th} term $= 2 \times (-3)^4 = 2 \times 81 = 162$

6^{th} term $= 2 \times (-3)^5 = 2 \times (-243) = -486$

Exercise 11B

Question 1.

Which term of the G.P. :

$-10, \frac{5}{\sqrt{3}}, -\frac{5}{6}, \ldots\ldots \ldots$ is $-\frac{5}{72}$?

Solution:

For the given G.P.:

First term, $a = -10$

Common ratio, $r = \dfrac{5/\sqrt{3}}{-10} = -\dfrac{1}{2\sqrt{3}}$

If $-\dfrac{5}{72}$ is the n^{th} term of the given G.P., then

$$-\frac{5}{72} = ar^{n-1}$$

$$\Rightarrow -\frac{5}{72} = -10 \times \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \frac{1}{144} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \frac{1}{2 \times 2 \times 2 \times 2 \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \left(\frac{1}{2\sqrt{3}}\right)^{4} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow n - 1 = 4$$

$$\Rightarrow n = 5$$

Question 2.

The fifth term of a G.P. is 81 and its second term is 24. Find the geometric progression.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

5^{th} term $= 81 \Rightarrow ar^4 = 81$

2^{nd} term $= 24 \Rightarrow ar = 24$

Now, $\dfrac{ar^4}{ar} = \dfrac{81}{24}$

$$\Rightarrow r^3 = \frac{27}{8}$$

$$\Rightarrow r = \frac{3}{2}$$

$ar = 24$

$\Rightarrow a = 16$

$\therefore$ G.P. $= a, ar, ar^2, ar^3, \ldots\ldots$

$$= 16, 24, 16 \times \left(\frac{3}{2}\right)^2, 16 \times \left(\frac{3}{2}\right)^3, \ldots..$$

$$= 16, 24, 36, 54, \ldots..$$

Question 3.

Fourth and seventh terms of a G.P. are $\frac{1}{18}$ and $-\frac{1}{486}$ respectively. Find the GP.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

4^{th} term $= \frac{1}{18} \Rightarrow ar^3 = \frac{1}{18}$

7^{th} term $= -\frac{1}{486} \Rightarrow ar^6 = -\frac{1}{486}$

Now, $\frac{ar^6}{ar^3} = \frac{-1/486}{1/18}$

$\Rightarrow r^3 = -\frac{1}{27}$

$\Rightarrow r = -\frac{1}{3}$

$ar^3 = \frac{1}{18}$

$\Rightarrow a \times \left(-\frac{1}{3}\right)^3 = \frac{1}{18}$

$\Rightarrow a = -\frac{27}{18} = -\frac{3}{2}$

$\therefore$ G.P. $= a, ar, ar^2, ar^3, \ldots\ldots$

$$= -\frac{3}{2}, -\frac{3}{2} \times \left(\frac{-1}{3}\right), -\frac{3}{2} \times \left(-\frac{1}{3}\right)^2, \frac{1}{18}, \ldots..$$

$$= -\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, \ldots..$$

Question 4.
If the first and the third terms of a G.P. are 2 and 8 respectively, find its second term.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

$\therefore$ 1^{st} term = a = 2

And, 3^{rd} term = 8 $\Rightarrow ar^2 = 8$

Now, $\frac{ar^2}{a} = \frac{8}{2}$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

When a = 2 and r = 2

2^{nd} term = ar = $2 \times 2 = 4$

When a = 2 and r = −2

2^{nd} term = ar = $2 \times (-2) = -4$

Question 5.
The product of 3rd and 8th terms of a G.P. is 243. If its 4^{th} term is 3, find its 7^{th} term.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

Now,

$t_3 \times t_8 = 243$

$\Rightarrow ar^2 \times ar^7 = 243$

$\Rightarrow a^2r^9 = 243$ ....(i)

Also,

$t_4 = 3$

$\Rightarrow ar^3 = 3$

$\Rightarrow a = \frac{3}{r^3}$

Substituting the value of a in (i), we get

$\left(\frac{3}{r^3}\right)^2 \times r^9 = 243$

$\Rightarrow \frac{9}{r^6} \times r^9 = 243$

$\Rightarrow r^3 = 27$

$\Rightarrow r = 3$

$\Rightarrow a = \frac{3}{3^3} = \frac{3}{27} = \frac{1}{9}$

$\therefore$ 7^{th} term $= t_7 = ar^6 = \frac{1}{9} \times (3)^6 = 81$

Question 6.
Find the geometric progression with 4^{th} term = 54 and 7^{th} term = 1458.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

4^{th} term $= 54 \Rightarrow ar^3 = 54$

7^{th} term $= 1458 \Rightarrow ar^6 = 1458$

Now, $\frac{ar^6}{ar^3} = \frac{1458}{54}$

$\Rightarrow r^3 = 27$

$\Rightarrow r = 3$

$ar^3 = 54$

$\Rightarrow a \times (3)^3 = 54$

$\Rightarrow a = \frac{54}{27} = 2$

$\therefore$ G.P. $= a,\ ar,\ ar^2,\ ar^3,\ \ldots\ldots$

$= 2,\ 2 \times 3,\ 2 \times (3)^2,\ 54,\ \ldots\ldots$

$= 2,\ 6,\ 18,\ 54,\ \ldots\ldots$

Question 7.

Second term of a geometric progression is 6 and its fifth term is 9 times of its third term. Find the geometric progression. Consider that each term of the G.P. is positive.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

Now, 2^{nd} term $= t_2 = 6 \Rightarrow ar = 6$

Also, $t_5 = 9 \times t_3$

$\Rightarrow ar^4 = 9 \times ar^2$

$\Rightarrow r^2 = 9$

$\Rightarrow r = \pm 3$

Since, each term of a G.P. is positive, we have $r = 3$

$ar = 6$

$\Rightarrow a \times 3 = 6 \Rightarrow a = 2$

$$\begin{aligned} \therefore \text{G.P.} &= a, ar, ar^2, ar^3, \ldots\ldots \\ &= 2, 6, 2 \times (3)^2, 2 \times (3)^3, \ldots\ldots \\ &= 2, 6, 18, 54, \ldots\ldots \end{aligned}$$

Question 8.

The fourth term, the seventh term and the last term of a geometric progression are 10, 80 and 2560 respectively. Find its first term, common ratio and number of terms.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

Now,

4^{th} term $= t_4 = 10 \Rightarrow ar^3 = 10$

7^{th} term $= t_7 = 80 \Rightarrow ar^6 = 80$

$$\frac{ar^6}{ar^3} = \frac{80}{10}$$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 2$

$ar^3 = 10$

$\Rightarrow a \times (2)^3 = 10$

$$\Rightarrow a = \frac{10}{8} = \frac{5}{4}$$

Last term = l = 2560

Let there be n terms in given G.P.

$\Rightarrow t_n = 2560$

$\Rightarrow ar^{n-1} = 2560$

$\Rightarrow \frac{5}{4} \times (2)^{n-1} = 2560$

$\Rightarrow (2)^{n-1} = 2048$

$\Rightarrow (2)^{n-1} = (2)^{11}$

$\Rightarrow n - 1 - 11$

$\Rightarrow n = 12$

Thus, we have

First term $= \frac{5}{4}$, Common ratio $= 2$ and Number of terms $= 12$

Question 9.

If the 4th and 9th terms of a G.P. are 54 and 13122 respectively, find the GP. Also, find its general term.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

Now,

4^{th} term $= t_4 = 54 \Rightarrow ar^3 = 54$

9^{th} term $= t_9 = 13122 \Rightarrow ar^8 = 13122$

$\frac{ar^8}{ar^3} = \frac{13122}{54}$

$\Rightarrow r^5 = 243$

$\Rightarrow r = 3$

$ar^3 = 54$

$\Rightarrow a \times (3)^3 = 54$

$\Rightarrow a = \frac{54}{27} = 2$

$\therefore$ Required G.P. $= a, ar, ar^2, ar^3, \ldots\ldots$

$= 2, 2 \times 3, 2 \times (3)^2, 54$

$= 2, 6, 18, 54$

General term $= t_n = ar^{n-1} = 2 \times (3)^{n-1}$

Question 10.
The fifth, eight and eleventh terms of a geometric progression are p, q and r respectively. Show that : $q^2 = pr$.

Solution:

Let the first term of the G.P. be a and its common ratio be r.

5^{th} term = $t_5 = p$
$\Rightarrow ar^4 = p$

8^{th} term = $t_8 = q$
$\Rightarrow ar^7 = q$

11^{th} term = $t_{11} = r$
$\Rightarrow ar^{10} = r$

Now,
$pr = ar^4 \times ar^{10} = a^2 \times r^{14} = (a \times r^7)^2 = q^2$
$\Rightarrow q^2 = pr$

Exercise 11C

Question 1.
Find the seventh term from the end of the series : $\sqrt{2}$, 2, $2\sqrt{2}$, ... 32.

Solution:

Given series: $\sqrt{2}, 2, 2\sqrt{2}, \ldots, 32$

Now, $\frac{2}{\sqrt{2}} = \sqrt{2}, \ \frac{2\sqrt{2}}{2} = \sqrt{2}$

So, the given series is a G.P. with common ratio, $r = \sqrt{2}$

Here, last term, $l = 32$

$\therefore 7^{th}$ term from an end $= \frac{l}{r^6} = \frac{32}{\left(\sqrt{2}\right)^6} = \frac{32}{8} = 4$

Question 2.
Find the third term from the end of the GP.

$\frac{2}{27}, \frac{2}{9}, \frac{2}{3}$, ………….. 162

Solution:

Given G.P.: $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots, 162$

Here,

Common ratio, $r = \frac{2/9}{2/27} = 3$

Last term, $l = 162$

$\therefore$ 3^{rd} term from an end $= \frac{l}{r^2} = \frac{162}{(3)^2} = \frac{162}{9} = 18$

Question 3.

For the $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}$, ……….. 81;
find the product of fourth term from the beginning and the fourth term from the end.

Solution:

Given G.P.: $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots\ldots, 81$

Here,

Common ratio, $r = \frac{1/9}{1/27} = 3$

First term, $a = \frac{1}{27}$ and Last term, $l = 81$

$\therefore$ 4^{th} term from the beginning $= ar^3 = \frac{1}{27} \times (3)^3 = \frac{1}{27} \times 27 = 1$

And, 4^{th} term from an end $= \frac{l}{r^3} = \frac{81}{(3)^3} = \frac{81}{27} = 3$

Thus, required product $= 1 \times 3 = 3$

Question 4.

If for a G.P., p^{th}, q^{th} and r^{th} terms are a, b and c respectively ; prove that :
$(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$

Solution:

Let the first term of the G.P. be A and its common ratio be R.

Then,

p^{th} term $= a \Rightarrow AR^{p-1} = a$

q^{th} term $= b \Rightarrow AR^{q-1} = b$

r^{th} term $= c \Rightarrow AR^{r-1} = c$

Now,

$$\begin{aligned} a^{q-r} \times b^{r-p} \times c^{p-q} &= (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q} \\ &= A^{q-r}.R^{(p-1)(q-r)} \times A^{r-p}.R^{(q-1)(r-p)} \times A^{p-q}.R^{(r-1)(p-q)} \\ &= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)} \\ &= A^0 \times R^0 \\ &= 1 \end{aligned}$$

Taking log on both the sides, we get

$\log\left(a^{q-r} \times b^{r-p} \times c^{p-q}\right) = \log 1$

$\Rightarrow (q-r)\log a + (r-p)\log b + (p-q)\log c = 0$ ....(proved)

Question 5.

If a, b and c in G.P., prove that : $\log a^n$, $\log b^n$ and $\log c^n$ are in A.P.

Solution:

Here, a, b, c are in G.P.

$\Rightarrow b^2 = ac$

Taking log on both sides, we get

$\log(b^2) = \log(ac)$

$\Rightarrow 2\log b = \log a + \log c$

$\Rightarrow \log b + \log b = \log a + \log c$

$\Rightarrow \log b - \log a = \log c - \log b$

$\Rightarrow \log a, \log b$ and $\log c$ are in A.P.

Question 6.
If each term of a G.P. is raised to the power x, show that the resulting sequence is also a G.P.

Solution:

Let $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ be a G.P. with common ratio r.

$\Rightarrow \frac{a_{n+1}}{a_n} = r$ for all $n \in N$

If each term of a G.P. is raised to the power x, we get the sequence

$a_1^x, a_2^x, a_3^x, \ldots\ldots, a_n^x, \ldots\ldots$

Now, $\frac{(a_{n+1})^x}{(a_n)^x} = \left(\frac{a_{n+1}}{a_n}\right)^x = r^x$ for all $n \in N$

Hence, $a_1^x, a_2^x, a_3^x, \ldots\ldots, a_n^x, \ldots\ldots$ is also a G.P.

Question 7.
If a, b and c are in A.P. a, x, b are in G.P. whereas b, y and c are also in G.P. Show that : x^2, b^2, y^2 are in A.P.

Solution:

a, b and c are in A.P.

$\Rightarrow 2b = a + c$

a, x and b are in G.P.

$\Rightarrow x^2 = ab$

b, y and c are in G.P.

$\Rightarrow y^2 = bc$

Now,

$$\begin{aligned} x^2 + y^2 &= ab + bc \\ &= b(a + c) \\ &= b \times 2b \\ &= 2b^2 \end{aligned}$$

$\Rightarrow x^2, b^2$ and y^2 are in A.P.

Question 8.

If a, b, c are in G.P. and a, x, b, y, c are in A.P., prove that :

(i) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$ (ii) $\frac{a}{x}+\frac{c}{y}=2$

Solution 8(i).

a, b and c are in G.P.

$\Rightarrow b^2 = ac$

a, x, b, y and c are in A.P.

$\Rightarrow 2x = a+b \Rightarrow x = \frac{a+b}{2}$

$2b = x+y \Rightarrow b = \frac{x+y}{2}$

$2y = b+c \Rightarrow y = \frac{b+c}{2}$

Now,

$$\frac{1}{x}+\frac{1}{y} = \frac{2}{a+b}+\frac{2}{b+c}$$
$$= \frac{2b+2c+2a+2b}{ab+ac+b^2+bc}$$
$$= \frac{2a+2c+4b}{ab+b^2+b^2+bc}$$
$$= \frac{2a+2c+4b}{ab+2b^2+bc}$$
$$= \frac{2(a+c+2b)}{b(a+2b+c)}$$
$$= \frac{2}{b}$$

Solution 8(ii).

a, b and c are in G.P.

$\Rightarrow b^2 = ac$

a, x, b, y and c are in A.P.

$\Rightarrow 2x = a+b \Rightarrow x = \frac{a+b}{2}$

$2b = x+y \Rightarrow b = \frac{x+y}{2}$

$2y = b+c \Rightarrow y = \frac{b+c}{2}$

Now,

$$\frac{a}{x}+\frac{c}{y}=\frac{2a}{a+b}+\frac{2c}{b+c}$$

$$=\frac{2a(b+c)+2c(a+b)}{(a+b)(b+c)}$$

$$=\frac{2ab+2ac+2ac+2bc}{ab+ac+b^2+bc}$$

$$=\frac{2ab+4ac+2bc}{ab+b^2+b^2+bc}$$

$$=\frac{2(ab+2ac+bc)}{ab+2b^2+bc}$$

$$=\frac{2(ab+2ac+bc)}{ab+2ac+bc}$$

$$=2$$

Question 9.

If a, b and c are in A.P. and also in G.P., show that: a = b = c.

Solution:

a, b and c are in A.P.

$\Rightarrow 2b = a + c$

$\Rightarrow b = \frac{a+c}{2}$

a, b and c are also in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac$

$\Rightarrow \frac{a^2+c^2+2ac}{4} = ac$

$\Rightarrow a^2 + c^2 + 2ac = 4ac$

$\Rightarrow a^2 + c^2 - 2ac = 0$

$\Rightarrow (a-c)^2 = 0$

$\Rightarrow a - c = 0$

$\Rightarrow a = c$

Now, $2b = a + c$

$\Rightarrow 2b = a + a$

$\Rightarrow 2b = 2a$

$\Rightarrow b = a$

Thus, we have $a = b = c$

Question 10.

The first term of a G.P. is a and its n^{th} term is b, where n is an even number. If the product of first n numbers of this G.P. is P ; prove that : $p^2 – (ab)^n$.

Solution:

For a G.P.,

First term = a

Let the common ratio = r

n^{th} term = b

$\Rightarrow ar^{n-1} = b$

P = Product of first n numbers of the given G.P.

$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times ar^{n-1}$

$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times b$

$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times \frac{b}{r^2} \times \frac{b}{r} \times b$

$\Rightarrow P = (ab) \times \left(ar \times \frac{b}{r}\right) \times \left(ar^2 \times \frac{b}{r^2}\right) \times \ldots\ldots\ldots \frac{n}{2}$ terms

$\Rightarrow P = (ab) \times (ab) \times (ab) \times \ldots\ldots\ldots \frac{n}{2}$ terms

$\Rightarrow P = (ab)^{\frac{n}{2}}$

$\Rightarrow P = \sqrt{ab^n}$

$\Rightarrow p^2 = ab^n$

Question 11.

If a, b, c and d are consecutive terms of a G.P. ; prove that :
$(a^2 + b^2)$, $(b^2 + c^2)$ and $(c^2 + d^2)$ are in GP.

Solution:

Let r be the common ratio of this G.P.

Given : a, b, c, d are in G.P.

$\Rightarrow 1^{st} = a,$

2^{nd} term $= b = ar,$

3^{rd} term $= c = ar^2$

4^{th} term $= d = ar^3$

Now, $\left(b^2 + c^2\right)^2 = \left[(ar)^2 + (ar^2)^2\right]^2$

$= \left[a^2r^2 + a^2r^4\right]^2$

$= \left[a^2r^2\left(1 + r^2\right)\right]^2$

$= a^4r^4\left(1 + r^2\right)^2$

And, $\left(a^2 + b^2\right) \times \left(c^2 + d^2\right) = \left[a^2 + (ar)^2\right] \times \left[(ar^2)^2 + (ar^3)^2\right]$

$= \left[a^2 + a^2r^2\right] \times \left[a^2r^4 + a^2r^6\right]$

$= a^2\left(1 + r^2\right) \times a^2r^4\left(1 + r^2\right)$

$= a^4r^4\left(1 + r^2\right)^2$

$\Rightarrow \left(b^2 + c^2\right)^2 = \left(a^2 + b^2\right) \times \left(c^2 + d^2\right)$

i.e. $\dfrac{b^2 + c^2}{a^2 + b^2} = \dfrac{c^2 + d^2}{b^2 + c^2}$

Hence, $\left(a^2 + b^2\right)$, $\left(b^2 + c^2\right)$ and $\left(c^2 + d^2\right)$ are in G.P.

Question 12.

If a, b, c and d are consecutive terms of a G.P. To prove:

$\dfrac{1}{a^2 + b^2}$, $\dfrac{1}{b^2 + c^2}$ **and** $\dfrac{1}{c^2 + d^2}$ **are in G.P.**

Solution:

Let r be the common ratio of this G.P.

Given : a, b, c, d are in G.P.

$\Rightarrow 1^{st} = a,$

2^{nd} term = b = ar,

3^{rd} term = c = ar^2

4^{th} term = d = ar^3

$$\text{Now, } \left(\frac{1}{b^2+c^2}\right)^2 = \left[\frac{1}{(ar)^2+(ar^2)^2}\right]^2$$

$$= \left[\frac{1}{a^2r^2+a^2r^4}\right]^2$$

$$= \frac{1}{a^4r^4}\left[\frac{1}{1+r^2}\right]^2$$

$$= \frac{1}{a^4r^4} \times \frac{1}{\left(1+r^2\right)^2}$$

$$\text{And, } \left(\frac{1}{a^2+b^2}\right) \times \left(\frac{1}{c^2+d^2}\right) = \left[\frac{1}{a^2+(ar)^2}\right] \times \left[\frac{1}{(ar^2)^2+(ar^3)^2}\right]$$

$$= \left[\frac{1}{a^2+a^2r^2}\right] \times \left[\frac{1}{a^2r^4+a^2r^6}\right]$$

$$= \frac{1}{a^2}\left(\frac{1}{1+r^2}\right) \times \frac{1}{a^2r^4}\left(\frac{1}{1+r^2}\right)$$

$$= \frac{1}{a^4r^4} \times \frac{1}{\left(1+r^2\right)^2}$$

$$\Rightarrow \left(\frac{1}{b^2+c^2}\right)^2 = \left(\frac{1}{a^2+b^2}\right) \times \left(\frac{1}{c^2+d^2}\right)$$

Hence, $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}$ and $\frac{1}{c^2+d^2}$ are in G.P.

Exercise 11D

Question 1.

Find the sum of G.P. :

(i) 1 + 3 + 9 + 27 + ……….. to 12 terms.

(ii) 0.3 + 0.03 + 0.003 + 0.0003 + …… to 8 terms.

(iii) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ to 9 terms.

(iv) $1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\ldots$ to n terms.

(v) $\frac{x+y}{x-y}+1+\frac{x-y}{x+y}+\ldots$ upto n terms.

(vi) $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots\ldots$ to n terms.

Solution 1(i).

Given G.P. : $1+3+9+27+\ldots$
Here,
first term, $a = 1$

common ratio, $r = \frac{3}{1} = 3\ (r > 1)$

number of terms to be added, $n = 12$

$$\therefore S_n = \frac{a(r^n - 1)}{r-1}$$

$$\Rightarrow S_{12} = \frac{1(3^{12}-1)}{3-1} = \frac{3^{12}-1}{2} = \frac{531441-1}{2} = \frac{531440}{2} = 265720$$

Solution 1(ii).

Given G.P. : $0.3+0.03+0.003+0.003+\ldots$
Here,
first term, $a = 0.3$

common ratio, $r = \frac{0.03}{0.3} = 0.1\ (r < 1)$

number of terms to be added, $n = 8$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{0.3(1-(0.1)^8)}{1-0.1} = \frac{0.3(1-(0.1)^8)}{0.9} = \frac{1-(0.1)^8}{3} = \frac{1}{3}\left(1-\frac{1}{10^8}\right)$$

Solution 1(iii).

Given G.P. : $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots\ldots$

Here,

first term, a = 1

common ratio, $r = \frac{-1/2}{1} = -\frac{1}{2}$ (r < 1)

number of terms to be added, n = 9

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{1\left(1-\left(-\frac{1}{2}\right)^9\right)}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{1-\left(-\frac{1}{2}\right)^9}{1+\frac{1}{2}}$$

$$= \frac{1+\frac{1}{2^9}}{\frac{3}{2}}$$

$$= \frac{2}{3}\left(1+\frac{1}{2^9}\right)$$

$$= \frac{2}{3}\left(1+\frac{1}{512}\right)$$

$$= \frac{2}{3} \times \frac{513}{512}$$

$$= \frac{171}{256}$$

Solution 1(iv).

Given G.P.: $1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\ldots..$ upto n terms

Here,

first term, $a = 1$

common ratio, $r = \frac{-1/3}{1} = -\frac{1}{3}$ $(r < 1)$

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1+\frac{1}{3}}$$

$$= \frac{\left[1-\left(-\frac{1}{3}\right)^n\right]}{\frac{4}{3}}$$

$$= \frac{3}{4}\left[1-\left(-\frac{1}{3}\right)^n\right]$$

Solution 1(v).

Given G.P.: $\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \ldots\ldots$ upto n terms

Here,

first term, $a = \frac{x+y}{x-y}$

common ratio, $r = \frac{1}{\frac{x+y}{x-y}} = \frac{x-y}{x+y}$ $(r < 1)$

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{1-\left(\frac{x-y}{x+y}\right)}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{x+y-x+y}{x+y}}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{2y}{x+y}}$$

$$= \frac{(x+y)^2\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{2y(x-y)}$$

Solution 1(vi).

Given G.P.: $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots\ldots$ upto n terms

Here,

first term, $a=\sqrt{3}$

common ratio, $r=\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}}=\frac{1}{3}\ (r<1)$

number of terms to be added = n

$$\therefore S_n=\frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n=\frac{\sqrt{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$

$$=\frac{\sqrt{3}\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$$

$$=\frac{3\sqrt{3}}{2}\left(1-\frac{1}{3^n}\right)$$

Question 2.

How many terms of the geometric progression 1+4 + 16 + 64 + ... must be added to get sum equal to 5461?

Solution:

Given G.P.: $1+4+16+64+\ldots\ldots$

Here,

first term, $a=1$

common ratio, $r=\frac{4}{1}=4\ (r>1)$

Let the number of terms to be added = n

Then, $S_n=5461$

$$\Rightarrow \frac{a(r^n-1)}{r-1}=5461$$

$$\Rightarrow \frac{1\left(4^n - 1\right)}{4-1} = 5461$$

$$\Rightarrow \frac{4^n - 1}{3} = 5461$$

$$\Rightarrow 4^n - 1 = 16383$$

$$\Rightarrow 4^n = 16384$$

$$\Rightarrow 4^n = 4^7$$

$$\Rightarrow n = 7$$

Hence, required number of terms = 7

Question 3.

The first term of a G.P. is 27 and its 8th term is $\frac{1}{81}$. Find the sum of its first 10 terms.

Solution:

Given,

First term, $a = 27$

8^{th} term $= ar^7 = \frac{1}{81}$

$n = 10$

Now,

$$\frac{ar^7}{a} = \frac{1/81}{27}$$

$$\Rightarrow r^7 = \frac{1}{2187}$$

$$\Rightarrow r^7 = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3}\ (r < 1)$$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_{10} = \frac{27\left(1 - \left(\frac{1}{3}\right)^{10}\right)}{1 - \frac{1}{3}}$$

$$= \frac{27\left(1 - \frac{1}{3^{10}}\right)}{\frac{2}{3}}$$

$$\frac{81}{2}\left(1 - \frac{1}{3^{10}}\right)$$

Question 4.

A boy spends ₹ 10 on first day, ₹ 20 on second day, ₹ 40 on third day and so on. Find how much, in all, will he spend in 12 days?

Solution:

Amount spent on 1^{st} day = Rs. 10

Amount spent on 2^{nd} day = Rs. 20

Amount spent on 3^{rd} day = Rs. 40 and so on

Now, $\frac{20}{10} = 2, \frac{40}{20} = 2,$

Thus, 10, 20, 40, ... is a G.P. with first term, $a = 10$

and common ratio, $r = 2\ (r > 1)$

$\therefore$ Total amount spent in 12 days = S_{12}

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{12} = \frac{10(2^{12} - 1)}{2 - 1} = 10(2^{12} - 1) = 10(4096 - 1) = 10 \times 4095 = 40950$$

Hence, the total amount spent in 12 days is Rs. 40950.

Question 5.

The 4th and the 7th terms of a G.P. are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of this G.P.

Solution:

For a G.P.,

4^{th} term = $ar^3 = \frac{1}{27}$

7^{th} term = $ar^6 = \frac{1}{729}$

Now, $\frac{ar^6}{ar^3} = \frac{1/729}{1/27}$

$$\Rightarrow r^3 = \frac{1}{27} = \left(\frac{1}{3}\right)^3$$

$$\Rightarrow r = \frac{1}{3}\ (r < 1)$$
$$\Rightarrow a \times \frac{1}{27} = \frac{1}{27}$$
$$\Rightarrow a = 1$$
$$S_n = \frac{a(1-r^n)}{1-r}$$
$$\Rightarrow S_n = \frac{1\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$
$$= \frac{\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$$
$$= \frac{3}{2}\left(1-\frac{1}{3^n}\right)$$

Question 6.

A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728 ; find its first term.

Solution:

For a G.P.,

Common ratio, $r = 3\ \ (r > 1)$

Last term, $l = 486$

$S = 728$

$$\Rightarrow \frac{lr - a}{r - 1} = 728$$
$$\Rightarrow \frac{486 \times 3 - a}{3 - 1} = 728$$
$$\Rightarrow \frac{1458 - a}{2} = 728$$
$$\Rightarrow 1458 - a = 1456$$

Hence, the first term is 2.

Question 7.
Find the sum of G.P. : 3, 6, 12, ... 1536.

Solution:

Given G.P.: 3, 6, 12, ..., 1536
Here,
First term, $a = 3$

Common ratio, $r = \frac{6}{3} = 2$ $(r > 1)$

Last term, $l = 1536$

$$\therefore \text{ Required sum} = \frac{lr - a}{r - 1}$$
$$= \frac{1536 \times 2 - 3}{2 - 1}$$
$$= 3072 - 3$$
$$= 3069$$

Question 8.
How many terms of the series 2 + 6 + 18 + ... must be taken to make the sum equal to 728 ?

Solution:

Given series: $2 + 6 + 18 + ...$

Now, $\frac{6}{2} = 3$, $\frac{18}{6} = 3$

Thus, given series is a G.P. with first term, $a = 2$
and common ratio, $r = 3$ $(r > 1)$
Let the number of terms to be added $= n$
Then, $S_n = 728$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 728$$
$$\Rightarrow \frac{2(3^n - 1)}{3 - 1} = 728$$
$$\Rightarrow 3^n - 1 = 728$$
$$\Rightarrow 3^n = 729$$
$$\Rightarrow 3^n = 3^6$$
$$\Rightarrow n = 6$$

Hence, required number of terms = 6

Question 9.
In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125 : 152.

Find its common ratio.

Solution:

Let a be the first term and r be the common ratio of given G.P.

Now, sum of first three terms $= S_3 = \dfrac{a(r^3-1)}{r-1}$

Now, sum of first six terms $= S_6 = \dfrac{a(r^6-1)}{r-1}$

It is given that

$$\frac{\frac{a(r^3-1)}{r-1}}{\frac{a(r^6-1)}{r-1}} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3-1}{(r^3)^2-(1)^2} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$$

$$\Rightarrow \frac{1}{r^3+1} = \frac{125}{152}$$

$$\Rightarrow r^3+1 = \frac{152}{125}$$

$$\Rightarrow r^3 = \frac{152}{125} - 1 = \frac{152-125}{125} = \frac{27}{125}$$

$$\Rightarrow r^3 = \left(\frac{3}{5}\right)^3$$

$$\Rightarrow r = \frac{3}{5}$$

Hence, the common ratio is $\frac{3}{5}$.

Question 10.

Find how many terms of G.P. $\frac{2}{9}-\frac{1}{3}+\frac{1}{2}$ .... must be added to get the sum equal to $\frac{55}{72}$?

Solution:

Given G.P.: $\frac{2}{9}-\frac{1}{3}+\frac{1}{2}$........

Here,

First term, $a=\frac{2}{9}$

Common ratio, $r=\frac{-1/3}{2/9}=-\frac{3}{2}<1$

Let required number of terms be n.

$\Rightarrow S_n=\frac{55}{72}$

$\Rightarrow \frac{a(1-r^n)}{1-r}=\frac{55}{72}$

$\Rightarrow \frac{\frac{2}{9}\left(1-\left(\frac{-3}{2}\right)^n\right)}{1-\left(-\frac{3}{2}\right)}=\frac{55}{72}$

$\Rightarrow \frac{\frac{2}{9}\left(1-\left(\frac{-3}{2}\right)^n\right)}{\frac{5}{2}}=\frac{55}{72}$

$\Rightarrow \frac{2}{9}\left(1-\left(\frac{-3}{2}\right)^n\right)=\frac{55}{72}\times\frac{5}{2}$

$\Rightarrow 1-\left(\frac{-3}{2}\right)^n=\frac{55}{72}\times\frac{5}{2}\times\frac{9}{2}$

$\Rightarrow 1-\left(\frac{-3}{2}\right)^n=\frac{275}{32}$

$\Rightarrow 1-\frac{275}{32}=\left(\frac{-3}{2}\right)^n$

$\Rightarrow -\frac{243}{32}=\left(\frac{-3}{2}\right)^n$

$\Rightarrow \left(-\frac{3}{2}\right)^5=\left(\frac{-3}{2}\right)^n$

$\Rightarrow n=5$

$\therefore$ Required number of terms = 5

Question 11.
If the sum $1 + 2 + 2^2 + \dots\dots\dots + 2^{n-1}$ is 255, find the value of n.

Solution:

Required series: $1+2+2^2+\dots\dots+2^{n-1}$

Now, $\frac{2}{1}=2,\ \frac{2^2}{2}=2$

Thus, given series is a G.P. with

first term, $a = 1$

common ratio, $r = 2\ (r > 1)$

Last term, $l = 2^{n-1}$

Let there be n terms in the series.

Then, $S_n = 255$

$\Rightarrow \frac{lr-a}{r-1}=255$

$\Rightarrow \frac{2^{n-1}\times 2-1}{2-1}=255$

$\Rightarrow 2^{n-1}\times 2-1=255$

$\Rightarrow 2^{n-1}\times 2=256$

$\Rightarrow 2^{n-1}=128$

$\Rightarrow 2^{n-1}=2^7$

$\Rightarrow n-1=7$

$\Rightarrow n=8$

Question 12.
Find the geometric mean between :

(i) $\frac{4}{9}$ and $\frac{9}{4}$
(ii) 14 and $\frac{7}{32}$
(iii) 2a and $8a^3$

Solution 12(i).

Geometric mean between $\frac{4}{9}$ and $\frac{9}{4}=\sqrt{\frac{4}{9}\times\frac{9}{4}}=\sqrt{1}=1$

Solution 12(ii).

Geometric mean between 14 and $\frac{7}{32}=\sqrt{14\times\frac{7}{32}}=\sqrt{\frac{49}{16}}=\frac{7}{4}=1\frac{3}{4}$

Solution 12(iii).

Geometric mean between 2a and $8a^3 = \sqrt{2a \times 8a^3} = \sqrt{16 \times a^4} = 4a^2$

Question 13.

The sum of three numbers in G.P. is $\frac{39}{10}$ and their product is 1. Find the numbers.

Solution:

Let the numbers be $\frac{a}{r}$, a and ar.

$\Rightarrow \frac{a}{r} \times a \times ar = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Now, $\frac{a}{r} + a + ar = \frac{39}{10}$

$\Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10}$

$\Rightarrow \frac{1 + r + r^2}{r} = \frac{39}{10}$

$\Rightarrow 10 + 10r + 10r^2 = 39r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (2r - 5)(5r - 2) = 0$

$\Rightarrow r = \frac{5}{2} \text{ or } r = \frac{2}{5}$

Thus, required terms are:

$\frac{a}{r}, a, ar = \frac{1}{5/2}, 1, 1 \times \frac{5}{2} \quad \text{OR} \quad \frac{1}{2/5}, 1, 1 \times \frac{2}{5}$

$= \frac{2}{5}, 1, \frac{5}{2} \quad \text{OR} \quad \frac{5}{2}, 1, \frac{2}{5}$

Question 14.

The first term of a G.P. is -3 and the square of the second term is equal to its 4^{th} term. Find its 7^{th} term.

Solution:

For a G.P.,
First term, $a = -3$
It is given that,
$(2^{nd}\ \text{term})^2 = 4^{th}\ \text{term}$
$\Rightarrow (ar)^2 = ar^3$
$\Rightarrow a^2r^2 = ar^3$
$\Rightarrow a = r$
$\Rightarrow r = -3$
Now, $7^{th}\ \text{term} = ar^6 = -3 \times (-3)^6 = -3 \times 729 = -2187$

Question 15.
Find the 5^{th} term of the G.P. $\frac{5}{2}$, 1, ...

Solution:
First term (a) = $\frac{5}{2}$

And, common ratio $(r) = \frac{1}{\frac{5}{2}} = \frac{2}{5}$

Now, $t_n = ar^{n-1}$
$\Rightarrow 5^{th}\ \text{term} = t_5 = \frac{5}{2} \times \left(\frac{5}{2}\right)^{5-1} = \frac{5}{2} \times \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$

Question 16.
The first two terms of a G.P. are 125 and 25 respectively. Find the 5th and the 6th terms of the G.P.

Solution:
First term (a) = 125

And, common ratio $(r) = \frac{25}{125} = \frac{1}{5}$
Now, $t_n = ar^{n-1}$
$\Rightarrow 5^{th}\ \text{term} = t_5 = 125 \times \left(\frac{1}{5}\right)^{5-1} = 125 \times \left(\frac{1}{5}\right)^4 = 125 \times \frac{1}{625} = \frac{1}{5}$
$\Rightarrow 6^{th}\ \text{term} = t_6 = 125 \times \left(\frac{1}{5}\right)^{6-1} = 125 \times \left(\frac{1}{5}\right)^5 = 125 \times \frac{1}{3125} = \frac{1}{25}$

Question 17.
Find the sum of the sequence $-\frac{1}{3}$, 1, – 3, 9, ... upto 8 terms.

Solution:

Here,

$$\frac{1}{-\frac{1}{3}} = \frac{-3}{1} = \frac{9}{-3} = -3$$

Thus, the given sequence is a G.P. with first term $(a) = -\frac{1}{3}$ and common ratio $(r) = -3$ $(r < 1)$.

Number of terms to be added, $n = 8$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{-\frac{1}{3}\left(1-(-3)^8\right)}{1+3} = \frac{-1+3^8}{12} = \frac{1}{12}(3^8 - 1)$$

Question 18.
The first term of a G.P. in 27. If the 8thterm be $\frac{1}{81}$, what will be the sum of 10 terms ?

Solution:

Given,

First term, $a = 27$

8^{th} term $= ar^7 = \frac{1}{81}$

$n = 10$

Now,

$$\frac{ar^7}{a} = \frac{1/81}{27}$$

$$\Rightarrow r^7 = \frac{1}{2187}$$

$$\Rightarrow r^7 = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3} \; (r < 1)$$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_{10} = \frac{27\left(1-\left(\frac{1}{3}\right)^{10}\right)}{1-\frac{1}{3}}$$

$$= \frac{27\left(1-\frac{1}{3^{10}}\right)}{\frac{2}{3}}$$

$$= \frac{81}{2}\left(1-\frac{1}{3^{10}}\right)$$

$$= \frac{81}{2}(1-3^{-10})$$

Question 19.
Find a G.P. for which the sum of first two terms is -4 and the fifth term is 4 times the third term.

Solution:

Let the five terms of the given G.P. be

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Given, sum of first two terms = -4

$$\frac{a}{r^2} + \frac{a}{r} = -4$$

$$\Rightarrow \frac{a+ar}{r^2} = -4$$

$$\Rightarrow a + ar = -4r^2$$

$$\Rightarrow a(1+r) = -4r^2$$

$$\Rightarrow a = -\frac{4r^2}{1+r}$$

And, 5^{th} term = $4(3^{rd}$ term)

$\Rightarrow ar^2 = 4(a)$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

When r = +2,

$$a = -\frac{4(2)^2}{1+2} = -\frac{16}{3}$$

When r = -2,

$$a = -\frac{4(-2)^2}{1-2} = 16$$

Thus, the required terms are $\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2.

i.e. $\frac{-\frac{16}{3}}{4}, \frac{-\frac{16}{3}}{2}, -\frac{16}{3}, -\frac{16}{3}\times 2, -\frac{16}{3}\times 4$ OR $\frac{16}{4}, \frac{16}{-2}, 16, 16(-2), 16\times 4$

i.e. $-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, -\frac{32}{3}, -\frac{64}{3}$ OR $4, -8, 16, -32, 64$

Additional Questions

Question 1.

Find the sum of n terms of the series :
(i) 4 + 44 + 444 + ...
(ii) 0.8 + 0.88 + 0.888 + ...

Solution 1(i).

$$\begin{aligned}
\text{Required sum} &= 4 + 44 + 444 + \ldots\ldots \text{ upto n terms} \\
&= 4(1 + 11 + 111 + \ldots\ldots \text{ upto n terms}) \\
&= \frac{4}{9}(9 + 99 + 999 + \ldots\ldots \text{upto n terms}) \\
&= \frac{4}{9}\left[(10-1) + (100-1) + (1000-1) + \ldots\ldots \text{ upto n terms}\right] \\
&= \frac{4}{9}\left[\begin{array}{l}(10 + 10^2 + 10^3 + \ldots\ldots \text{upto n terms}) \\ \quad - (1 + 1 + 1 + \ldots\ldots \text{upto n terms})\end{array}\right] \\
&= \frac{4}{9}\left[\frac{10(10^n - 1)}{10-1} - n\right] \\
&= \frac{4}{9}\left[\frac{10}{9}(10^n - 1) - n\right]
\end{aligned}$$

Solution 1(ii).

$$\text{Required sum} = 0.8+0.88+0.888+\ldots\ldots\ \text{upto n terms}$$
$$= 8(0.1+0.11+0.111+\ldots\ldots\ \text{upto n terms})$$
$$= \frac{8}{9}(0.9+0.99+0.999+\ldots\ldots\ \text{upto n terms})$$
$$= \frac{8}{9}\left[(1-0.1)+(1-0.01)+(1-0.001)+\ldots\ldots\ \text{upto n terms}\right]$$
$$= \frac{8}{9}\left[\begin{array}{l}(1+1+1+\ldots\ldots\text{upto n terms})\\ \qquad -(0.1+0.01+0.001+\ldots\ldots\text{upto n terms})\end{array}\right]$$
$$= \frac{8}{9}\left[\begin{array}{l}(1+1+1+\ldots\ldots\text{upto n terms})\\ \qquad -\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots\ldots\text{upto n terms}\right)\end{array}\right]$$
$$= \frac{8}{9}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right] \qquad \left[\because r=\frac{1}{10}<1\right]$$
$$= \frac{8}{9}\left[n-\frac{10}{9}\times\frac{1}{10}\left(1-\frac{1}{10^n}\right)\right]$$
$$= \frac{8}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$$

Question 2.
Find the sum of infinite terms of each of the following geometric progression:

(i) $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\ldots$

(ii) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots\ldots$

(iii) $\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\ldots\ldots$

(iv) $\sqrt{2}-\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}-\frac{1}{4\sqrt{2}}+\ldots\ldots$

(v) $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\frac{1}{9\sqrt{3}}+\ldots\ldots$

Solution 2(i).

Given G.P. : $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\dots\dots$

Here,

First term, $a=1$

Common ratio, $r=\frac{1/3}{1}=\frac{1}{3}\quad\left(|r|=\left|\frac{1}{3}\right|=\frac{1}{3}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$

Solution 2(ii).

Given G.P. : $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\dots\dots$

Here,

First term, $a=1$

Common ratio, $r=\frac{-1/2}{1}=-\frac{1}{2}\quad\left(|r|=\left|-\frac{1}{2}\right|=\frac{1}{2}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{1}{1-\left(-\frac{1}{2}\right)}=\frac{1}{1+\frac{1}{2}}=\frac{2}{3}$

Solution 2(iii).

Given G.P. : $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\dots$

Here,

First term, $a=\frac{1}{3}$

Common ratio, $r=\frac{1/3^2}{1/3}=\frac{1}{3}\quad\left(|r|=\left|\frac{1}{3}\right|=\frac{1}{3}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$

Solution 2(iv).

Given G.P. : $\sqrt{2}-\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}-\frac{1}{4\sqrt{2}}+\ldots\ldots$

Here,

First term, $a=\sqrt{2}$

Common ratio, $r=\frac{-1/\sqrt{2}}{\sqrt{2}}=-\frac{1}{2}\left(|r|=\left|-\frac{1}{2}\right|=\frac{1}{2}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{\sqrt{2}}{1-\left(-\frac{1}{2}\right)}=\frac{\sqrt{2}}{1+\frac{1}{2}}=\frac{2\sqrt{2}}{3}$

Solution 2(v).

Given G.P. : $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}-\frac{1}{9\sqrt{3}}+\ldots\ldots$

Here,

First term, $a=\sqrt{3}$

Common ratio, $r=\frac{1/\sqrt{3}}{\sqrt{3}}=\frac{1}{3}\left(|r|=\left|\frac{1}{3}\right|=\frac{1}{3}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{\sqrt{3}}{1-\frac{1}{3}}=\frac{\sqrt{3}}{\frac{2}{3}}=\frac{3\sqrt{3}}{2}$

Question 3.
The second term of a G.P. is 9 and sum of its infinite terms is 48. Find its first three terms.

Solution:

Let a be the first term and r be the common ratio of a G.P.

2^{nd} term, $t_2=ar=9$

$\Rightarrow r=\frac{9}{a}$

Sum of its infinite terms, $S=48$

$\Rightarrow \frac{a}{1-r}=48$

$$\Rightarrow \frac{a}{1-\frac{9}{a}} = 48$$

$$\Rightarrow \frac{a^2}{a-9} = 48$$

$$\Rightarrow a^2 = 48a - 432$$

$$\Rightarrow a^2 - 48a + 432 = 0$$

$$\Rightarrow a^2 - 36a - 12a + 432 = 0$$

$$\Rightarrow a(a-36) - 12(a-36) = 0$$

$$\Rightarrow (a-36)(a-12) = 0$$

$$\Rightarrow a = 36 \text{ or } a = 12$$

When $a = 36$, $r = \frac{9}{36} = \frac{1}{4}$

$\Rightarrow 1^{st}$ term $= 36$,

2^{nd} term $= ar = 36 \times \frac{1}{4} = 9$

3^{rd} term $= ar^2 = 36 \times \frac{1}{16} = \frac{9}{4}$

When $a = 12$, $r = \frac{9}{12} = \frac{3}{4}$

$\Rightarrow 1^{st}$ term $= 12$,

2^{nd} term $= ar = 12 \times \frac{3}{4} = 9$

3^{rd} term $= ar^2 = 12 \times \frac{9}{16} = \frac{27}{4}$

Question 4.

Find three geometric means between $\frac{1}{3}$ and 432.

Solution:

Let G_1, G_2, G_3 be three geometric means between $a = \frac{1}{3}$ and $b = 432$.

Then, $\frac{1}{3}, G_1, G_2, G_3, 432$ is a G.P.

Thus, we have

First term $= a = \frac{1}{3}$

5^{th} term of the G.P. $= ar^4 = 432$

$\Rightarrow \frac{1}{3} \times r^4 = 432$

$\Rightarrow r^4 = 1296$

$\Rightarrow r^4 = 6^4$

$\Rightarrow r = 6$

$\therefore G_1 = ar = \frac{1}{3} \times 6 = 2$

$G_2 = ar^2 = \frac{1}{3} \times 6 \times 6 = 12$

$G_3 = ar^3 = \frac{1}{3} \times 6 \times 6 \times 6 = 72$

Question 5.

Find :
(i) two geometric means between 2 and 16
(ii) four geometric means between 3 and 96.
(iii) five geometric means between $3\frac{5}{9}$ and $40\frac{1}{2}$

Solution 5(i).

Let G_1, G_2 be two geometric means between $a = 2$ and $b = 16$.

Then, $2, G_1, G_2, 16$ is a G.P.

Thus, we have

First term $= a = 2$

4^{th} term of the G.P. $= ar^3 = 16$

$\Rightarrow 2 \times r^3 = 16$

$\Rightarrow r^3 = 8$

$\Rightarrow r^3 = 2^3$

$\Rightarrow r = 2$

$\therefore G_1 = ar = 2 \times 2 = 4$

$G_2 = ar^2 = 2 \times 2 \times 2 = 8$

Solution 5(ii).

Let G_1, G_2, G_3, G_4 be four geometric means between $a = 3$ and $b = 96$.

Then, $3, G_1, G_2, G_3, G_4, 96$ is a G.P.

Thus, we have

First term $= a = 3$

6^{th} term of the G.P. $= ar^5 = 96$

$\Rightarrow 3 \times r^5 = 96$

$\Rightarrow r^5 = 32$

$\Rightarrow r^5 = 2^5$

$\Rightarrow r = 2$

$\therefore\ G_1 = ar = 3 \times 2 = 6$

$G_2 = ar^2 = 3 \times 4 = 12$

$G_3 = ar^3 = 3 \times 8 = 24$

$G_4 = ar^4 = 3 \times 16 = 48$

Solution 5(iii).

Let G_1, G_2, G_3, G_4, G_5 be five geometric means between

$a = 3\frac{5}{9} = \frac{32}{9}$ and $b = 40\frac{1}{2} = \frac{81}{2}$.

Then, $\frac{32}{9}, G_1, G_2, G_3, G_4, G_5, \frac{81}{2}$ is a G.P.

Thus, we have

First term $= a = \frac{32}{9}$

7^{th} term of the G.P. $= ar^6 = \frac{81}{2}$

$\Rightarrow \frac{32}{9} \times r^6 = \frac{81}{2}$

$\Rightarrow r^6 = \frac{81}{2} \times \frac{9}{32}$

$\Rightarrow r^6 = \frac{729}{64}$

$\Rightarrow r^6 = \left(\frac{3}{2}\right)^6$

$\Rightarrow r = \frac{3}{2}$

$$\therefore G_1 = ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$$

$$G_2 = ar^2 = \frac{32}{9} \times \frac{9}{4} = 8$$

$$G_3 = ar^3 = \frac{32}{9} \times \frac{27}{8} = 12$$

$$G_4 = ar^4 = \frac{32}{9} \times \frac{81}{16} = 18$$

$$G_5 = ar^5 = \frac{32}{9} \times \frac{243}{32} = 27$$

Question 6.

The sum of three numbers in G.P. is $\frac{39}{10}$ and their product is 1. Find the numbers.

Solution:

Sum of three numbers in G.P. = $\frac{39}{10}$ and their product = 1

Let number be $\frac{a}{r}$, a, ar, then

$$\frac{a}{r} \times a \times ar = 1 \Rightarrow a^3 = 1 = (1)^3$$

$$\therefore a = 1$$

$$\text{and } \frac{a}{r} + a + ar = \frac{39}{10}$$

$$\Rightarrow a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$$

$$\frac{1}{r} + 1 + r = \frac{39}{10} \times 1 = \frac{39}{10}$$

$$r + \frac{1}{r} = \frac{39}{10} - 1 = \frac{39-10}{10} = \frac{29}{10}$$

$$r^2 + 1 = \frac{29}{10} r$$

$$10r^2 + 10 = 29r \Rightarrow 10r^2 - 29r + 10 = 0$$
$$\Rightarrow 10r^2 - 4r - 25r + 10 = 0$$
$$\Rightarrow 2r(5r - 2) - 5(5r - 2) = 0$$
$$\Rightarrow (5r - 2)(2r - 5) = 0$$

Either $5r - 2 = 0$, then $r = \frac{2}{5}$

or $2r - 5 = 0$, then $r = \frac{5}{2}$

$\therefore$ Numbers are $\frac{2}{5}, 1, \frac{4}{25}$, or $\frac{5}{2}, 1, \frac{25}{4}$

Question 7.
Find the numbers in G.P. whose sum is 52 and the sum of whose product in pairs is 624.

Solution:

Let the numbers be a, ar and ar^2.

$\Rightarrow a + ar + ar^2 = 52 \quad \ldots.(i)$

And, $(a \times ar) + (ar \times ar^2) + (ar^2 \times a) = 624$

$\Rightarrow a^2r + a^2r^3 + a^2r^2 = 624$

$\Rightarrow ar(a + ar^2 + ar) = 624$

$\Rightarrow ar \times 52 = 624 \quad \ldots.[From (i)]$

$\Rightarrow ar = 12$

$\Rightarrow a = \frac{12}{r}$

Substituting in (i), we get

$\frac{12}{r} + \frac{12}{r} \times r + \frac{12}{r} \times r^2 = 52$

$\Rightarrow \frac{12}{r} + 12 + 12r = 52$

$\Rightarrow \frac{12 + 12r + 12r^2}{r} = 52$

$\Rightarrow 12 + 12r + 12r^2 = 52r$

$\Rightarrow 12r^2 - 40r + 12 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$
$\Rightarrow 3r(r-3) - 1(r-3) = 0$
$\Rightarrow (3r-1)(r-3) = 0$
$\Rightarrow r = \frac{1}{3}$ or $r = 3$
$\Rightarrow a = \frac{12}{1/3} = 36$ or 4

Thus, required terms are:

$a,\ ar,\ ar^2 = 36,\ 36 \times \frac{1}{3},\ 36 \times \frac{1}{9}$ OR $4,\ 4 \times 3,\ 4 \times 9$
$= 36,\ 12,\ 4$ OR $4,\ 12,\ 36$

Question 8.

The sum of three numbers in G.P. is 21 and the sum of their squares is 189. Find the numbers.

Solution:

Let the numbers be a, ar and ar^2.
$\Rightarrow (a)^2 + (ar)^2 + (ar^2)^2 = 189$
$\Rightarrow a^2 + a^2r^2 + a^2r^4 = 189$
And, $a + ar + ar^2 = 21$
$\Rightarrow (a + ar + ar^2)^2 = 21^2$
$\Rightarrow a^2 + a^2r^2 + a^2r^4 + 2a^2r + 2a^2r^3 + 2a^2r^2 = 441$
$\Rightarrow 189 + 2ar(a + ar^2 + ar) = 441$
$\Rightarrow 2ar \times 21 = 441 - 189$
$\Rightarrow 42ar = 252$
$\Rightarrow ar = 6$
$\Rightarrow r = \frac{6}{a}$
Now, $a + ar + ar^2 = 21$
$\Rightarrow a + a \times \frac{6}{a} + a \times \frac{36}{a^2} = 21$
$\Rightarrow a + 6 + \frac{36}{a} = 21$
$\Rightarrow a^2 + 6a + 36 = 21a$
$\Rightarrow a^2 - 15a + 36 = 0$
$\Rightarrow a^2 - 12a - 3a + 36 = 0$

$\Rightarrow a(a-12)-3(a-12)=0$

$\Rightarrow (a-12)(a-3)=0$

$\Rightarrow a=12 \quad \text{or} \quad a=3$

$\Rightarrow r=\frac{6}{12}=\frac{1}{2} \quad \text{or} \quad r\frac{6}{3}=2$

Thus, required terms are:

$a,\ ar,\ ar^2 = 12,\ 12\times\frac{1}{2},\ 12\times\frac{1}{4} \quad \text{OR} \quad 3,\ 3\times 2,\ 3\times 4$

$= 12,\ 6,\ 3 \quad \text{OR} \quad 3,\ 6,\ 12$